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Time-dependent spacetimes in AdS/CFT: bubble and black hole

Simon F. Ross and Georgina Titchener

Centre for Particle Theory, Department of Mathematical Sciences
University of Durham
South Road, Durham DH1 3LE, U.K.
E-mail: S.F.Ross@durham.ac.uk, G.L.Titchener@durham.ac.uk

Abstract: We extend the study of time-dependent backgrounds in the AdS/CFT correspondence by examining the relation between bulk and boundary for the smooth 'bubble of nothing' solution and for the locally AdS black hole which has the same asymptotic geometry. These solutions are asymptotically locally AdS, with a conformal boundary conformal to de Sitter space cross a circle. We study the cosmological horizons and relate their thermodynamics in the bulk and boundary. We consider the α-vacuum ambiguity associated with the de Sitter space, and find that only the euclidean vacuum is well-defined on the black hole solution. We argue that this selects the euclidean vacuum as the preferred state in the dual strongly coupled CFT.

Keywords: Superstring Vacua, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The study of time-dependent backgrounds in string theory is a key area of development of the theory. An understanding of dynamical spacetimes is essential for many applications of current interest, such as cosmological evolution or black hole evaporation. Consideration of more general spacetime backgrounds can also illuminate new aspects of string theory, just as the consideration of quantum field theory on more general spacetime backgrounds brought to light new effects such as Unruh and Hawking radiation, and offered a new perspective on what the essential elements of quantum field theory are.

The Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence [1, 2] is a promising approach to the understanding of dynamical spacetime in string theory, since the dual field theory description is fully non-perturbative, offering a description which in principle encompasses both the dynamics of the background spacetime and the behaviour of strings or fields propagating in this dynamical background. However, our understanding of the correspondence for dynamical spacetimes remains very patchy. The aim in this paper is to extend this dictionary by exploring aspects of the relation between bulk and boundary for two interesting time-dependent asymptotically locally AdS solutions.

The first solution we are interested in is the 'bubble of nothing' solution in anti-de Sitter space. This solution was introduced in [3, 4], following the analysis of [5] of asymptotically flat bubble of nothing solutions as time-dependent backgrounds in string theory. These solutions are constructed by the double analytic continuation of black hole solutions, and describe a spacetime with a compact circle direction which shrinks to zero size on a surface which expands exponentially in the non-compact directions. This kind of solution was originally introduced in [6] to describe an instability of the Kaluza-Klein vacuum. The new observation of [5] is that they also provide nice examples of time-dependent backgrounds for string theory, as they are smooth vacuum solutions, which exhibit many of the issues we would like to address in the study of time-dependence, such as particle creation and non-trivial vacuum ambiguities for quantum fields. In [3, 4], an asymptotically locally AdS bubble was constructed by double analytic continuation of the Schwarzschild-AdS black hole solution. In [4], the counterterm subtraction procedure was used to obtain the stress tensor of the dual field theory.

At large distances, this 'bubble of nothing' solution approaches a locally AdS spacetime. This locally AdS spacetime is in fact just a quotient of AdS, and was interpreted previously in [7, 8] as a black hole solution. That is, this is the higher-dimensional analogue of the BTZ black hole [9, 10]. This is a time-dependent solution: there is no Killing vector which is timelike everywhere outside the event horizon. Although it is not as smooth as the bubble of nothing solution, this is clearly an interesting example of a time-dependent geometry in its own right, and we will see that it has some interesting properties.

These two solutions have the same asymptotics, so they should be related to different states in the same field theory on the asymptotic boundary. This boundary is a de Sitter space cross a circle: the circle corresponds to the direction that is compactified in the locally AdS black hole, and which degenerates at the bubble in the bubble of nothing solution. Our aim is to explore the properties of the spacetimes, and relate them to the dual field theory. We will focus on understanding the relation between horizons in the spacetime and the boundary theory, and considering the question of choices of vacuum state in the bulk and the boundary.

In the next section, we give a brief review of the two solutions. We then discuss the horizons in these solutions in section 3. We show that the solutions have Killing horizons which can be interpreted as cosmological or acceleration horizons respectively, and which are in both cases naturally related to the de Sitter cosmological horizon in the boundary geometry. In particular, we show that the entropy of the bulk and boundary horizons agree if we introduce a cutoff at large radius. This sets up a novel correspondence relating horizons in the bulk and boundary, as opposed to relating horizons in the bulk to thermal states in the boundary. We argue that the black hole event horizon, on the other hand, does not have a thermodynamic interpretation.

Thus, the thermodynamic properties in the bulk are identified with the thermodynamic properties of the de Sitter horizon in the boundary theory. We are proposing that the time-dependence of the bulk spacetime is completely encoded in the time-dependence of the boundary spacetime, and the appropriate state in the dual CFT is simply some natural vacuum state on this curved, time-dependent background. These examples are thus a particularly simple context for further investigations of time-dependence in string theory, since we have reduced the problem to the more well-understood one of studying quantum field theory in a time-dependent background.

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One of the most interesting features of time-dependent spacetimes is that they do not have unique vacuum states. We would therefore like to use our solutions as a laboratory for studying the description of vacuum ambiguities in AdS/CFT. The natural vacuum ambiguity to consider in this context is the α-vacua in de Sitter space, since the boundary has a de Sitter factor, and there are coordinates in the bulk which write these spaces in de Sitter slicings. It has been known for some time [11]–[14] that de Sitter space has a one-parameter family of vacuum states invariant under the de Sitter isometry group, called α-vacua. There is a unique member of this family which has the same short-distance singularity as in flat space [13, 15], which is also the euclidean vacuum obtained by analytic continuation from the sphere [16, 17]. There has nonetheless been considerable controversy in the literature about whether the additional de Sitter-invariant vacua are physical, particularly focusing on the definition of an interacting theory [18]–[29]. It is not our intention to address any of the issues raised in this literature; instead, we want to consider a different approach, using the behaviour of the analogues of the α-vacua in a free scalar field theory in the bulk spacetime to obtain information about α-vacua in the strongly-coupled dual field theory.

In section 4, we show that there are natural analogues of the α-vacuum ambiguity in these two bulk spacetimes, which we identify with the ability to choose α-vacua in the boundary theory. In section 5, we find that the propagators for the α-vacua on the locally AdS black hole have additional singularities at the event horizon of the black hole. We show that these additional singularities are reflected in a breakdown of the procedure of [30] for constructing the expectation value for the stress tensor. We also comment that the analytic continuation procedure used in [31, 32] to probe the region behind the event horizon can only be extended to the locally AdS black hole if we take the euclidean vacuum. We therefore argue that the analogues of α-vacua are not good vacuum states for the locally AdS black hole, since they break down on the event horizon.

We interpret this as evidence that the α-vacua are not good states for the strongly-coupled CFT on the boundary, which is de Sitter space cross a circle. Although the α-vacua appear to be acceptable states at least at the free level for the bubble of nothing solution, this spacetime only exists when the size of the circle is less than a maximum value. Thus, at least for a range of parameters, there is no obvious spacetime interpretation for the α-vacua in the CFT, whereas there is a spacetime interpretation for the euclidean vacuum. Thus, the euclidean vacuum is selected as a preferred state in the strongly-coupled CFT.

We summarize and present some concluding remarks in section 6.

2. Review of bubble & black hole solutions

The bubble of nothing solution is obtained by analytic continuation of the 5d Schwarzschild-AdS black hole,[1]

$$ds^2 = -\left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right) dt^2 + \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 \left(d\vartheta^2 + \cos^2\vartheta d\Omega_2^2\right), \qquad (2.1)$$

[1] We focus on the case of AdS_5 for definiteness, but it is a simple exercise to extend our remarks to other AdS_d with $d \geq 4$.

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the two-sphere. By analytically continuing two variables, $t \to i\chi$ and $\vartheta \to i\tau$, a novel solution of gravity with negative cosmological constant is found:

$$ds^2 = \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right) d\chi^2 + \left(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 \left[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)\right] . \tag{2.2}$$

We see that the proper length of the spacelike χ direction goes to zero at $r = r_+$, where r_+ is the root of $f(r) = l^2 r^2 + r^4 - r_0^2 l^2$,

$$r_+^2 = \frac{l^2}{2}\left[-1 + \sqrt{1 + \frac{4r_0^2}{l^2}}\right] . \tag{2.3}$$

To make the spacetime smooth at $r = r_+$, the coordinate χ must be identified periodically, with period

$$\Delta\chi = \frac{2\pi l^2 r_+}{2r_+^2 + l^2} . \tag{2.4}$$

There is no region of spacetime inside the surface $r = r_+$; this is the 'bubble of nothing'. Since the metric on this bubble is three-dimensional de Sitter space with scale r_+, we see that the bubble expands exponentially. At very early times, the region 'excised' is a very large sphere. As $\tau < 0$ increases, the size of the bubble shrinks to a minimum at $\tau = 0$. After this it grows again exponentially.

As was pointed out in [4], the solution (2.2) is asymptotically locally AdS: that is, at large distances, it approaches

$$ds^2 = \left(1 + \frac{r^2}{l^2}\right) d\chi^2 + \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 \left[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)\right] , \tag{2.5}$$

which is a locally AdS spacetime. This spacetime is the result of quotienting AdS by a boost isometry to make the coordinate χ periodic. This coordinate system is related to embedding coordinates for AdS by

$$\begin{aligned} x^1 &= (r^2 + l^2)^{1/2} \cosh\frac{\chi}{l} , \\ x^2 &= r \sinh\tau , \\ x^3 &= (r^2 + l^2)^{1/2} \sinh\frac{\chi}{l} , \\ x^4 &= r \cosh\tau \, \sin\theta \, \sin\phi , \\ x^5 &= r \cosh\tau \, \sin\theta \, \cos\phi , \\ x^6 &= r \cosh\tau \cos\theta , \end{aligned} \tag{2.6}$$

where x^μ are embedding coordinates in terms of which AdS is defined by $-(x^1)^2 - (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2 + (x^6)^2 = -l^2$. If χ is allowed to run over all values (2.6) provides a coordinatization of a part of AdS. Making χ periodic with some arbitrary period $\Delta\chi$ thus introduces discrete identifications on AdS along a boost isometry. As is evident from the form of the metric (2.5), the quotient preserves an $\mathrm{SO}(1,1) \times \mathrm{SO}(1,3)$ subgroup of the original $\mathrm{SO}(2,4)$ isometry group. Note that we are free to choose any period $\Delta\chi$ we wish in this quotient geometry.

As was stressed in [33], this spacetime has been studied previously [7, 8], as the higher-dimensional analogue of the BTZ black hole. It was also discussed in the recent classification of quotients of anti-de Sitter spaces [34]. It describes an interesting non-stationary black hole solution with a single exterior region. The structure of this solution is more easily understood by passing to a 'Kruskal' coordinate system [8],

$$\begin{aligned}
x^1 &= l\frac{1+y^2}{1-y^2}\cosh\frac{\chi}{l}\,, \\
x^2 &= 2l\frac{y^0}{1-y^2}\,, \\
x^3 &= l\frac{1+y^2}{1-y^2}\sinh\frac{\chi}{l}\,, \\
x^4 &= 2l\frac{y^1}{1-y^2}\,, \\
x^5 &= 2l\frac{y^2}{1-y^2}\,, \\
x^6 &= 2l\frac{y^3}{1-y^2}\,,
\end{aligned} \tag{2.7}$$

where we have written $y^2 = y^\mu y_\mu = -(y^0)^2 + (y^1)^2 + (y^2)^2 + (y^3)^2$. In terms of these coordinates, the radial coordinate of (2.5) is

$$r = l\frac{2\sqrt{y^2}}{1-y^2}\,, \tag{2.8}$$

and the coordinates (τ, θ, ϕ) in (2.5) parametrise the hyperboloids $y^2 = \text{constant}$. The metric in these Kruskal coordinates is

$$ds^2 = \frac{4l^2}{(1-y^2)^2}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + \frac{(1+y^2)^2}{(1-y^2)^2}d\chi^2\,. \tag{2.9}$$

One of the great advantages of this coordinate system is that it writes the metric at a constant χ as conformal to a flat space, enabling us to easily picture the causal structure of the spacetime. We see that the singularity at $r = 0$ in (2.5) is just a coordinate singularity, corresponding to the light cone of the origin, $y^2 = 0$ in the new coordinates. At $y^2 = -1$, the χ circle becomes null, and beyond this surface it will be timelike, so the quotient introduces closed timelike curves in this region. Following [7, 8], we assume this region of closed timelike curves is removed from the spacetime. Then $y^2 = -1$ becomes a singularity, since timelike curves end on it. The surface at $y^2 = 0$ then becomes an event horizon for the spacetime; observers who cross it will inevitably hit the singularity. The asymptotic boundary of the spacetime is at $y^2 = 1$. The geometry is depicted in figure 1. These Kruskal coordinates cover the whole spacetime.

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This is the natural higher-dimensional analogue of the BTZ black hole, but it clearly has a somewhat different global structure: the maximally extended spacetime described by (2.9) has only a single exterior region, with a connected asymptotic boundary. Furthermore, the global event horizon at $y^2 = 0$ is not a Killing horizon for any Killing vector, and one can easily see that the area of its cross-sections increase with time. We will discuss the horizons in this solution and their interpretation further in the next section.

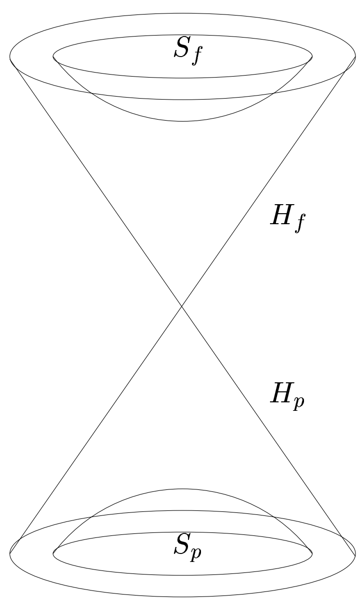

Figure 1: Three dimensions of the AdS black hole spacetime: one sphere direction and the S^1 factor are suppressed. The direction of increasing y_0 is up. S_f, S_p are the future and past singularities; H_f, H_p are the future and past horizons.

To understand the relation of these asymptotically locally AdS spacetimes to a dual field theory, we need to understand the conformal boundary of these spacetimes. Adopting a conformal factor $\Omega = l/r$, we see that the boundary metric for (2.2) is

$$ds^2_\Sigma = d\chi^2 + l^2 \left[-d\tau^2 + \cosh^2 \tau (d\theta^2 + \sin^2 \theta d\phi^2) \right] . \quad (2.10)$$

We can obtain the same result from (2.9) (in those coordinates $\Omega = (1-y^2)/2\sqrt{y^2}$). The dual CFT thus lives in a space which is three dimensional de Sitter space cross a circle, $\mathrm{dS}_3 \times S^1$. There is a single dimensionless parameter characterising this boundary geometry, the ratio of the radius of the circle to the size of the de Sitter factor, $\Delta\chi/l$. This is thus the physical information we can specify from the field theory point of view, and it will determine the bulk geometry.

For the locally AdS black hole, there is a unique bulk geometry for each choice of $\Delta\chi$. For the bubble of nothing, on the other hand, the value of r_0 characterising the bulk geometry is determined by solving (2.4), which does not give a one-to-one map between $\Delta\chi$ and r_0. There is a maximum value of $\Delta\chi$ for which this equation has a solution, $\Delta\chi_{max} = \sqrt{2}\pi l$, attained when $r_+^2 = l^2/2$, that is, when $r_0 = l/2$. If we choose $\Delta\chi$ greater than this maximum value, there is no corresponding bubble of nothing solution in the bulk. If we choose $\Delta\chi$ less than the maximum, there will be two solutions, with a smaller and a larger value for r_0.[2] It was argued in [4] that the solution with the smaller value of r_0 will be both classically and quantum mechanically unstable, and should therefore be disregarded, while we expect the other solution to be stable.

In [4], the boundary stress tensor for this solution was computed, using the counterterm subtraction procedure [35]–[37]. If we use the bubble of nothing as the bulk solution, we

[2]Note also that the χ circle is contractible in the bulk for the bubble of nothing solution, which fixes the bulk spin structure to be antiperiodic around this circle. Thus, this bulk geometry will only contribute to the path integral when we take antiperiodic boundary conditions for the fermions on this circle in the boundary. There is no such restriction for the locally AdS black hole solution.

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obtain a boundary energy density

$$\rho_{\text{bubble}} = -\frac{1}{16\pi G l^3}\left(r_0^2 + \frac{l^2}{4}\right) = -\frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right). \tag{2.11}$$

This calculation can also be applied to obtain the boundary stress tensor when we use the locally AdS black hole as the bulk solution, by setting $r_0 = 0$ in the previous result [33]. Thus, for this case

$$\rho_{\text{bh}} = -\frac{1}{64\pi G l} = -\frac{N^2}{32\pi^2 l^4}. \tag{2.12}$$

We note that the energy of the bubble is lower than the energy of the black hole, so it is possible for the black hole to decay into (some excited state on) the bubble of nothing. It was also pointed out in [4] that the stress tensor obtained for the black hole corresponds precisely to a geometrical contribution associated with the curvature of the background. This can presumably be interpreted as a reflection of the fact that the solution is simply a quotient of global AdS, so it corresponds to the dual field theory in a vacuum state where there is no state-dependent contribution to the stress-energy.

3. Horizons & thermodynamics

We now consider the relation between horizons and thermodynamics. The relation between the bulk black hole horizon and the thermal behaviour of the field theory on the boundary for the ordinary BTZ and Schwarzschild-AdS black hole solutions was one of the first things to be understood in the context of the AdS/CFT correspondence [38, 39]. The black hole solution described above has a global event horizon, but as noted previously, this is not a Killing horizon, and it is not clear if it should have a thermodynamic interpretation.

Both these spacetimes do however have Killing horizons in them, associated with the timelike Killing vectors that generate the worldlines of comoving observers in the coordinates of (2.2), (2.5). If we consider a given comoving observer, say the one at $\theta = 0$, then the corresponding Killing vector is

$$K = \cos\theta\partial_\tau + \tanh\tau\sin\theta\partial_\theta \tag{3.1}$$

and the Killing horizon where this Killing vector becomes null is at

$$\tanh\tau = \pm\cos\theta\,. \tag{3.2}$$

Note that the location of this Killing horizon is independent of r in these coordinates, and corresponds precisely to the usual Killing cosmological horizons in the de Sitter factor. These horizons are illustrated in figure 2.

In the bubble of nothing solutions, these Killing horizons correspond to cosmological horizons, just as they do in de Sitter space. It was noted in [5] that the bubble of nothing solution constructed from the Schwarzschild black hole has cosmological horizons, as the exponential expansion of the bubble prevents any one observer from seeing the whole of the spacetime. The same is true in the AdS bubble of nothing (2.2). Any observer's trajectory

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will asymptotically approach a constant coordinate position on the two-sphere, because of the exponential growth of the two-sphere's proper volume. The trajectory will thus lie within the region bounded by the corresponding Killing horizon, and that Killing horizon will be the cosmological horizon for this observer.

In the locally AdS black hole solution, these horizons are more naturally identified as acceleration horizons. Any observer who chooses to remain outside the black hole will asymptotically approach constant values of the angular coordinates, and the corresponding Killing horizon will be an event horizon for this observer by the same argument. However, it is clear from (2.9) that these observers are accelerating-uniformly accelerating if they remain at constant values of r. The Killing vector (3.1) is also, in these coordinates, simply a boost:

$$K = y^0 \partial_{y^3} + y^3 \partial_{y^0} \,. \qquad (3.3)$$

Thus, in the locally AdS black hole solution, we can think of these horizons as analogous to the Rindler horizons in flat space.

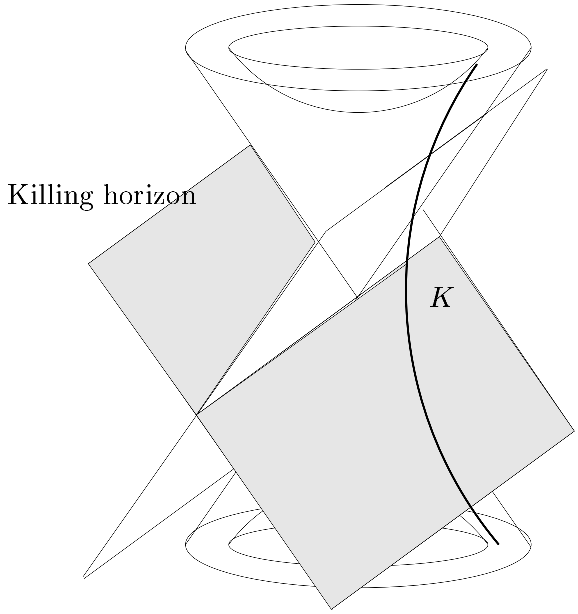

Figure 2: A timelike orbit of the Killing vector $K = \cos\theta\partial_\tau + \tanh\tau\sin\theta\partial_\theta$ and the plane where it becomes null, $\tanh\tau = \pm\cos\theta$.

What we would like to explain now is the interpretation of these horizons in the dual field theory. The novelty here is that the horizons are non-compact, and intersect the asymptotic boundary. This will imply a rather different relation between the horizons and the dual: the horizon in the bulk is related to a horizon in the boundary, rather than to thermal effects from considering a non-trivial mixed state in the boundary field theory. Since the structure of this cosmological/acceleration horizon is very similar in these two spacetimes, we will treat them together.

First, let us consider the thermal properties of the state. Since the horizon (3.2) is a Killing horizon, there is a unique regular (i.e., Hadamard) vacuum invariant under the action of this Killing vector. It will be a thermal state with respect to the notion of time translation defined by this Killing vector [40]. This state is clearly the usual euclidean vacuum, obtained by analytic continuation from the euclidean versions of (2.2), (2.5). The thermal properties of this state have a natural interpretation from the boundary point of view: also in the boundary, we have a cosmological horizon at (3.2), and the natural CFT vacuum state defined by analytic continuation from the euclidean version of the boundary geometry will look thermal from the point of view of comoving observers. That is, the bulk state is identified with a vacuum state in the CFT, and looks thermal simply because the CFT lives in a time-dependent background, a de Sitter universe (cross a circle).

However, there is a fly in the ointment for this very natural interpretation: we have two bulk geometries. How do we understand the difference between them from the boundary point of view? It seems that the circle plays the crucial role here. In the bubble of nothing solution, the fermions must be antiperiodic on this circle, whereas in the locally AdS black hole, we are free to choose either spin structure. We suggest that the bubble of nothing is related to a CFT vacuum with antiperiodic boundary conditions on the fermions, while the locally AdS black hole can be related to a CFT vacuum with periodic boundary conditions on the fermions.[3] This is analogous to the identification of global AdS with the NS ground state and the M=0 BTZ black hole with the RR ground state in the usual AdS_3 story [2]. Note however that in our case neither state is supersymmetric, as the background $dS_3 \times S^1$ geometry breaks all supersymmetry. As evidence in support of this suggestion, we note that the difference in energy between the bubble (2.11) and the black hole (2.12) goes like $1/\Delta\chi^4$ for small $\Delta\chi$, which is the expected behaviour for the Casimir energy associated with such a change in boundary conditions for fermions. The greatest problem with this suggestion is that the bubble of nothing solution only exists for $\Delta\chi \leq \Delta\chi_{max}$. We have no interpretation to suggest for this restriction, which seems very unnatural from the CFT point of view.

Having proposed a relation between thermal properties of the states in bulk and boundary, we would like to go on to make a more controversial suggestion, that there should also be an entropy associated with these horizons, by showing it has a natural interpretation in the boundary theory. One might think that this stands little chance of working, since the area of the bulk horizon is infinite, so the entropy $S = A/4G$ would also be infinite. How can we give an interpretation for this infinite entropy in terms of the boundary theory? However, this is precisely the right answer from the boundary point of view: the horizon in the bulk should be related to the cosmological horizon in the boundary. This has finite area, since it is just the usual de Sitter horizon, but the boundary theory is not coupled to gravity. We can formally include a gravitational term with $G = 0$, so the entropy for this horizon is indeed infinite.

To make a quantitative comparison, we introduce a cut off at $r = R$. The entropy of the bulk horizon inside this surface is

$$S_{\rm bulk} = \frac{A}{4G} = \frac{1}{4G}\int_{r_i}^{R} r\,dr \int d\phi d\chi\,, \tag{3.4}$$

where A is the area of the horizon's bifurcation surface $\tau = 0, \theta = \pi/2$, which we have written out explicitly in terms of an integration over (r, ϕ, χ). The lower limit of integration $r_i = r_+$ in the bubble of nothing solution and $r_i = 0$ in the locally AdS black hole, and $G = G_5$ is the gravitational constant in the bulk.

In the boundary theory, the introduction of the cut off at $r = R$ corresponds to an ultraviolet cutoff on the field theory, and introduces a coupling of the field theory to gravity, given by calculating the induced Einstein action obtained from integrating the bulk action

[3] Since it admits both spin structures, the black hole can also contribute when we consider antiperiodic boundary conditions. In that context, it is presumably interpreted as an excited state above the vacuum described by the bubble solution. It is only when we consider antiperiodic boundary conditions that the black hole can decay into a bubble.

over the radial direction. The action in the bulk is

$$I = \frac{1}{16\pi G} \int \sqrt{-g} d^5 x R \,. \tag{3.5}$$

If we set

$$ds^2 = f(r) d\chi^2 + f(r)^{-1} dr^2 + \frac{r^2}{l^2} \widehat{ds}^2 \tag{3.6}$$

we can rewrite this action as

$$I = \frac{1}{16\pi G} \int \sqrt{-\hat{g}} \frac{r^3}{l^3} dr d^4 x \frac{l^2}{r^2} \hat{R} + \cdots \,, \tag{3.7}$$

where $\hat{R}$ is the curvature of the three-dimensional metric $\widehat{ds}^2$, and ... denotes terms involving $f(r)$. This allows us to read off the Newton's constant in four dimensions as

$$\frac{1}{G_4} = \frac{1}{lG} \int_{r_i}^{R} r dr \,. \tag{3.8}$$

The entropy of the bifurcation surface $\tau = 0, \theta = \pi/2$ in the boundary theory is

$$S = \frac{A}{4G_4} = \frac{1}{4G_4} l \int d\phi d\chi \,, \tag{3.9}$$

which then agrees precisely with (3.4). Thus, we see that the entropy of the horizons in bulk and boundary agrees quantitatively.

Thus, the AdS/CFT correspondence identifies the entropy of the bulk cosmological/acceleration horizon with the entropy of the de Sitter horizon in the boundary. This calculation gives a powerful argument that even for non-compact horizons, the area should be regarded as an entropy, as argued in [41, 42].

One final point to note concerning the cosmological horizons is that there is a finite difference in entropy between the bubble of nothing solution and the locally AdS black hole: the entropy of the bubble of nothing is less than that of the black hole by

$$\Delta S = \frac{2\pi r_+^2 \Delta\chi}{4G} \,, \tag{3.10}$$

because of the different ranges of integration. This difference was absorbed in a change in the induced Newton's constant in the boundary from the point of view of the cutoff CFT discussed above, but it would be interesting to see if it could be related to some difference in the corresponding states, perhaps in the un-cutoff CFT.

Finally, what about the global event horizon in the locally AdS black hole? Should there be some entropy associated with this horizon as well? We will argue that the answer is no. First, we note that there is no independent temperature associated with this horizon. An observer outside the black hole will see a thermal bath, but this will come to them from the acceleration horizon that bounds the region of spacetime they can see, and not from the event horizon, which lies behind this acceleration horizon. Secondly, the event horizon is not a special surface in a spacelike slice of the spacetime: it is not, for instance, the boundary of a region of trapped surfaces. Indeed, as noticed previously, the event horizon in this spacetime is rather like the light cone of the origin in flat space. It is a purely teleological event horizon, which is a boundary of the past of infinity because of something that is going to happen in the future: the 'singularity' at $y^2 = -1$. The light cone of the origin in flat space can similarly form part of an event horizon, if a collapsing shell of matter is converging on it. However, we do not think we would associate an entropy with this horizon before the shell crossed it.

Therefore, we think the appropriate generalisation from the three-dimensional BTZ black hole, where the Killing horizon and event horizon are the same thing, is to associate an entropy with the Killing horizon in the locally AdS black hole in higher dimensions, and not to associate any entropy with the event horizon in this solution.

4. α-vacua in bubble and black hole

In the previous section, we have focused on relating the properties of the euclidean vacuum state on the bulk spacetimes to the dual CFT. However, one of the most interesting features of time-dependent spacetimes is that they do not have unique vacuum states. We would therefore like to use our solutions as a laboratory for studying the description of vacuum ambiguities in AdS/CFT.

The natural vacuum ambiguity to consider in this context is the α-vacua in de Sitter space. It has been known for some time [11]–[14] that de Sitter space has a one-parameter family of vacuum states invariant under the de Sitter isometry group. Our boundary geometry, which is de Sitter cross a circle, will clearly inherit this ambiguity, and we would now like to relate it to the bulk spacetime.

4.1 Review of α-vacua in de Sitter

To begin, we review the α-vacuum ambiguity in de Sitter space. Let G_E be the Wightman function for a massive scalar field ϕ on three dimensional de Sitter space obtained by analytically continuing the unique Wightman function on the euclidean sphere to 3d de Sitter. Then we define the euclidean vacuum state $|E\rangle$ by

$$G_E(x, x') = \langle E|\phi(x)\phi(x')|E\rangle \,. \tag{4.1}$$

We can choose a mode expansion for a free massive scalar field ϕ

$$\phi(x) = \sum_n \left(a_n \phi_n^E(x) + a_n^\dagger \phi_n^{E*}(x) \right) , \tag{4.2}$$

such that the vacuum state obeys

$$a_n|E\rangle = 0 \,. \tag{4.3}$$

The Wightman function can be re-written in terms of these modes as

$$G_E(x, x') = \sum_n \phi_n^E(x)\phi_n^{E*}(x') \,. \tag{4.4}$$

We can choose the positive frequency euclidean modes such that

$$\phi_n^E(x_A) = \phi_n^{E*}(x)\,, \tag{4.5}$$

where x_A is the point antipodal to x in de Sitter space. If x has coordinates θ, ϕ and τ, then x_A has coordinates $\pi - \theta, \phi + \pi, -\tau$.

The α-vacua are defined by observing that for any $\alpha \in \mathbb{C}$ with Re $\alpha < 0$, we can define a new mode expansion by the Bogoliubov transform [13, 14]

$$\tilde{\phi}_n(x) = N_\alpha(\phi_n^E(x) + e^\alpha \phi_n^{E*}(x)), \quad N_\alpha = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}\,. \tag{4.6}$$

The operators a_n must also be transformed, and the new operators are given by

$$\tilde{a}_n = N_\alpha(a_n^E - e^{\alpha*} a_n^{E\dagger})\,. \tag{4.7}$$

A new de Sitter invariant vacuum state $|\alpha\rangle$ is then defined by

$$\tilde{a}_n|\alpha\rangle = 0 \quad \forall\, n > 0 \tag{4.8}$$

and the Wightman function in this vacuum is

$$G_\alpha(x, x') = \langle\alpha|\tilde{\phi}(x)\tilde{\phi}(x')|\alpha\rangle = \sum_n \tilde{\phi}_n(x)\tilde{\phi}_n^*(x')\,. \tag{4.9}$$

This new propagator can be expressed in terms of the original euclidean modes and α. It is [43]

$$\begin{aligned} G_\alpha(x, x') \;=\; N_\alpha^2 \sum \Big[& \phi_n(x)\phi_n^*(x') + e^{\alpha+\alpha^*}\phi_n(x')\phi_n^*(x) + \\ & + e^{\alpha^*}\phi_n(x)\phi_n^*(x'_A) + e^\alpha \phi_n(x_A)\phi_n^*(x') \Big]\,. \end{aligned} \tag{4.10}$$

As a function of euclidean Wightman propagators, it can be written as

$$G_\alpha(x, x') = N_\alpha^2 \left(G_E(x, x') + e^{\alpha+\alpha^*} G_E(x', x) + e^{\alpha^*} G_E(x, x'_A) + e^\alpha G_E(x_A, x') \right). \tag{4.11}$$

These new vacua are automatically invariant under the continuous SO$(1, 3)$ symmetry of the de Sitter space, as can be seen from the above relation between α and euclidean propagators. If we take α to be real, the α-vacuum is also invariant under time reversal, which interchanges the last two terms in (4.11).

Unlike the euclidean vacuum, the α-vacuum is not thermal. In [44, 43], the departure from thermality was studied by considering the behaviour of a particle detector in the α-vacuum. One considers a monopole detector coupled to the scalar field ϕ by the interaction

$$g \int dt \phi(x(t)) m(t)\,, \tag{4.12}$$

where $x(t)$ is the path followed by the particle detector, with proper time t, and $m(t)$ is an operator acting on internal states of the detector. If the operator $m(t)$ has eigenstates $|E_i\rangle$ with energies E_i, we define the matrix elements m_{ij} as

$$m_{ij} = \langle E_i|m(0)|E_j\rangle\,. \tag{4.13}$$

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Then the probability that the detector reports a change in energy from E_i to E_j, $E_j > E_i$, is

$$P(E_i \to E_j) = g^2 |m_{ij}|^2 \int_{-\infty}^{\infty} dt dt' e^{-i(E_j - E_i)(t'-t)} G(x(t'), x(t)) \,, \tag{4.14}$$

where $G(x, x')$ is the Wightman function. Substituting in (4.10), [44, 43] found

$$\frac{P(E_i \to E_j)}{P(E_j \to E_i)} = e^{-2\pi\Delta E} \left| \frac{1 + e^{\alpha + \pi\Delta E}}{1 + e^{\alpha - \pi\Delta E}} \right|^2 \tag{4.15}$$

showing that the detector has a non-thermal response.

Note that in particular, at high energies the detector response becomes independent of the energy difference. This is a sign of the bad short-distance behaviour of the α-vacua. From (4.11), one can see that the short-distance singularity of the α-vacuum Wightman function is related to the singularity in the euclidean Wightman function by a factor of $N_\alpha^2 (1 + e^{\alpha + \alpha *})$. Thus, as $x \to x'$,

$$G_\alpha(x, x') = N_\alpha^2 \left[1 + e^{\alpha + \alpha^*} \right] \frac{1}{(2\pi)^2 \sigma(x, x')} + \cdots \,, \tag{4.16}$$

where $\sigma(x, x')$ is half the square of the geodesic distance between x and x', and $\ldots$ denotes less singular terms. The unusual coefficient of the singularity implies that the α-vacua are not Hadamard states: they have a different short-distance singularity than the flat-space vacuum propagator.

4.2 Vacuum ambiguity in bubble and black hole

Since the spacetimes we are interested in have a de Sitter factor, they will naturally all have a similar vacuum ambiguity. That is, if we consider a massive scalar field on either the bubble or the black hole, we can again choose a mode expansion satisfying

$$\phi_n^E(x_A) = \phi_n^{E*}(x) \,, \tag{4.17}$$

where x_A is the opposite point to x only in respect to the de Sitter factor coordinates. If x has full coordinates χ, r, τ, θ and ϕ, then x_A has coordinates $\chi, r, -\tau, \theta + \pi$, and $\phi + \pi$. Then we can define Bogoliubov transforms on the euclidean modes of this five-dimensional space in an identical way to eq. (4.6), to obtain new modes $\tilde{\phi}_n(x)$ and similarly from the operators on this space, a_n^E, we can define $\tilde{a}_n$. As a consequence there are also multiple vacua in this space, parametrised by α, satisfying $\tilde{a}_n |\alpha\rangle = 0$. The associated Wightman functions G_α defined using these new vacua or mode expansions can again be expressed in terms of the euclidean Wightman function on the bubble, G_E, by (4.11).[4] This α-vacuum ambiguity in the bulk is clearly related to a corresponding α-vacuum ambiguity in the boundary.

For the bubble of nothing solution (and of course for the boundary spacetime), the de Sitter coordinates of (2.2) cover the whole spacetime, and the de Sitter factor does not degenerate anywhere. The physics of these α-vacua is hence little different from the

[4]For the locally AdS black hole, it is possible to have such a vacuum ambiguity despite the fact that AdS has a unique invariant vacuum state because the quotient broke the SO(2, 4) isometry group to SO(1, 1) $\times$ SO(1, 3), which is no longer sufficient to determine a unique vacuum state.

familiar discussion in de Sitter space, and we will not elaborate on it here. The situation is more interesting for the locally AdS black hole, however, so we pursue this case in more detail.

First, we note that in the black hole background, we can find G_E explicitly. This is done by using the form of the propagator in full AdS$_d$ [45]. In AdS, the invariance under SO(2,4) implies that the propagator can only be a function of the geodesic distance between the two points x, x', $2\sqrt{\sigma(x,x')}$. It is in fact convenient to express the propagator as a function of $P = X \cdot X'$, where X, X' are the corresponding points in the embedding coordinates and the inner product is with respect to the metric of signature $(--++++)$ in the embedding space. This is related to geodesic distance through $\sigma(x,x') = -(P+1)$. In terms of P, the propagator is the solution to

$$\frac{1}{(P^2-1)^{\frac{d-2}{2}}}\partial_P\left((P^2-1)^{\frac{d}{2}}\partial_P G\right) - m^2 G = 0 \tag{4.18}$$

which is regular at $P = \infty$. The required function G is

$$G_E(P) = \frac{-e^{i\pi d/2}\sqrt{\pi}\Gamma(2b)}{2^{c-\frac{1}{2}}\Gamma(c)}(P^{-2})^b \;{}_2F_1\left(b+\frac{1}{2}, b\ ; c\ ; P^{-2}\right), \tag{4.19}$$

where

$$b = \frac{-1}{4} + \frac{1}{4}d + \frac{1}{4}\sqrt{(d-1)^2+4m^2}\,, \qquad c = 1 + \frac{1}{2}\sqrt{(d-1)^2+4m^2}\,. \tag{4.20}$$

It is divergent for $P = 0$ and for all $|P| = 1$ [46]. Those points which are light-like separated have $P = -1$.

To write the propagator for the quotiented space, we must sum over the images due to the periodic identification in the χ direction. The euclidean Wightman function on the locally AdS black hole can then be expressed as

$$G_E(P(x,x')) \sim \sum_{n=-\infty}^{\infty} P(x,{x'}_n)^{-2b} \;{}_2F_1\left((b+\frac{1}{2}), b\ ; c\ ; P(x,{x'}_n)^{-2}\right), \tag{4.21}$$

where the n dependence of x' indicates that we have included every image of x' under the identification.

This knowledge of the euclidean Wightman function is sufficient to determine the α-vacua Wightman functions, by using the formula (4.11) relating it to an expression with four terms, each involving the euclidean propagator. Recall that two of these depend only on x and x', but two involve the points antipodal to the original positions.

5. Singularities in the α-vacuum on the black hole

The important property of the α-vacuum Wightman function for the locally AdS black hole is that it develops new singularities on (and inside) the event horizon. These arise because the antipodal points, which were always separated by the cosmological horizon in de Sitter space, become causally related. In the coordinates of (2.9), the antipode of a point y^μ, χ is the point $-y^\mu, \chi$, and on and inside the light cone $y^2 = 0$, these points are causally separated. This means that on the horizon, there are additional short-distance singularities in the propagator: in the expression

$$G_\alpha(x, x') = N_\alpha^2 (G_E(x, x') + e^{\alpha+\alpha^*} G_E(x', x) + e^{\alpha^*} G_E(x, x'_A) + e^\alpha G_E(x_A, x')) \,, \qquad (5.1)$$

the last two terms can produce an additional singularity as $x \to x'$ if $x = x'$ is a point on the event horizon, as shown in figure 3. Because of the antipodal map involved, this additional singularity will have a completely different structure from the usual short-distance singularity: it is not simply proportional to $1/\sigma(x, x')$. It is thus potentially more dangerous than the previous failure of the α-vacuum to be Hadamard in de Sitter space.

We will argue that these singularities are a sign that the α-vacua are unphysical on this black hole spacetime. We will see below that they lead to a breakdown in the stress tensor on the event horizon, and obstruct attempts to probe the region behind the horizon by analytic continuation.

They are also interesting from the point of view of understanding the locally AdS black hole, as they are a clear sign that the event horizon is a special place in the black hole geometry. We think of these new singularities as analogous to the breakdown of a state with the wrong temperature or a non-thermal state on the event horizon of a Schwarzschild black hole, which selects the usual Hartle-Hawking state as the unique regular vacuum [40].

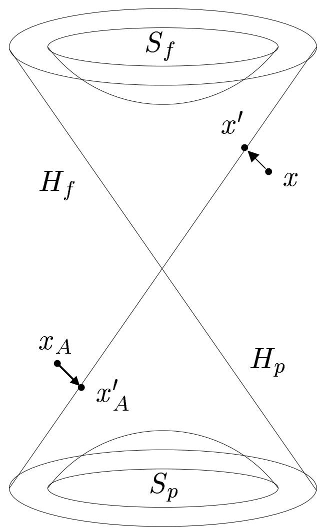

Figure 3: As $x \to x'$ on the event horizon, extra singularities appear due to the lightlike separation of x and x'_A.

5.1 Particle detectors

We now seek signs of these new singularities in the Wightman function in physical observables. We will first examine whether a particle detector crossing the horizon sees anything special, following the calculation reviewed in section 4.1. As before, the probability that the detector reports a change in energy from E_i to $E_j, E_j > E_i$, is

$$P(E_i \to E_j) = g^2 |m_{ij}|^2 \int_{-\infty}^{\infty} dt dt' e^{-i(E_j - E_i)(t'-t)} G_\alpha(x(t'), x(t)) \,. \qquad (5.2)$$

We can consider different trajectories for the detector. If we take the detector to stay at a constant r in (2.5), staying outside the black hole, then the behaviour in an α-vacuum will be the same as the behaviour in the de Sitter case, and the ratio of transition probabilities will be given by (4.15).

Consider instead an inertial observer, who freely falls across the black hole horizon following some geodesic. Consider first the euclidean vacuum. Then since $G_E(t' - t)$ is holomorphic in the lower half plane, we can close the contour of integration in (5.2) in the LHP to find that the probability is zero. This is the expected result, since the black hole solution is locally AdS, and the euclidean vacuum is obtained by sum over images from the usual AdS vacuum. To calculate the detector response in the α vacuum, we consider for definiteness a geodesic through the origin, so that $x_A(t) = x(-t)$ [5]. Then, using the expression (4.11) for the α-vacuum propagator in terms of the euclidean propagator, the detector response (5.2) is

$$\begin{aligned} P_\alpha(E_i \to E_j) = &\int_{-\infty}^{\infty} dt dt' e^{-i(E_j - E_i)(t'-t)} N_\alpha^2 \times \\ &\times \left(G_E(t,t') + e^{\alpha+\alpha^*} G_E(t',t) + e^{\alpha} G_E(-t,t') + e^{\alpha^*} G_E(t,-t') \right). \end{aligned} \tag{5.3}$$

In the last two terms, where the Wightman function involved is a function only of $t+t'$, we can immediately perform the integral over $t-t'$ by closing the contour in the LHP to get 0, showing that these terms make no contribution. The first term is merely proportional to the original integral in the euclidean vacuum, so it too gives no contribution.

The remaining term involves $G_E(t',t) = G_E(t'-t) = G_E(-t,-t')$. Thus, we have

$$P_\alpha(E_i \to E_j) = \int_{-\infty}^{\infty} dt dt' e^{-i(E_j - E_i)(t'-t)} N_\alpha^2 e^{\alpha+\alpha^*} G_E(-t,-t'). \tag{5.4}$$

Upon changing variables from t, t' to $-t, -t'$, the exponential picks up a minus sign, and we are left with the euclidean rate for the detector to see a change in energies from E_j to E_i,

$$P_\alpha(E_i \to E_j) = N_\alpha^2 e^{\alpha+\alpha^*} P_E\left(E_j \to E_i\right). \tag{5.5}$$

Similarly, if we consider the probability for the transition from $E_j \to E_i$ in the α-vacuum, the only contribution will come from the first term in (4.11), giving[6]

$$\begin{aligned} P_\alpha(E_j \to E_i) &= \int_{-\infty}^{\infty} dt dt' e^{-i(E_i - E_j)(t'-t)} N_\alpha^2 G_E(x(t'), x(t)) \\ &= N_\alpha^2 P_E(E_j \to E_i). \end{aligned} \tag{5.6}$$

The ratio of the two rates is

$$\frac{P_\alpha\left(E_i \to E_j\right)}{P_\alpha\left(E_j \to E_i\right)} = e^{\alpha+\alpha^*}. \tag{5.7}$$

We note that this result is independent of the energies involved, which might seem a disturbing result, but this is the usual problem with the short-distance structure of the α-vacuum, corresponding precisely to the high energy behaviour of (4.15). There is no sign in this calculation of the additional singularities at the event horizon, because the relevant parts of the α-vacuum propagator made no contribution to the calculation. This is perhaps surprising; if there is a breakdown in the quantum state on the horizon, we would expect the behaviour of a particle detector crossing the horizon to be affected.

[5] The geodesic used is not important, as the argument essentially relies only on the fact that the euclidean propagator depends only on the difference in proper time along the path, which is true for any geodesic in AdS.

[6] These rates are divergent because of the integration over $t + t'$, but this simply gives an overall factor that will cancel when considering the ratio of rates.

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5.2 Stress-energy tensor

Another natural observable to consider in looking for reflections of this new singularity in the propagator on the horizon is the expectation value of the stress tensor in the α-vacuum, $\langle T_{\mu\nu}\rangle_\alpha$. However, the usual construction of this quantity relies on the assumption that the state is Hadamard (see [47] for a review). As we have already noted, the α-vacua are not Hadamard states. Hence, we cannot define a stress tensor by the normal procedure in an α-vacuum state on any spacetime, and it therefore does not appear to be available to us as a probe of the new singularities in the state on the horizon of the locally AdS black hole.

Fortunately, Bernard and Folacci [30] have overcome this obstacle and defined a renormalised expectation value for the stress tensor in the α-vacua of de Sitter space despite the non-Hadamard form of the short-distance singularity.

To briefly review, we usually construct the stress tensor by taking a coincidence limit of an appropriate differential operator acting on the bi-distribution:

$$\langle T_{\mu\nu}(x)\rangle_\alpha = \lim_{x\to x'} \mathcal{D}_{\mu\nu} F(x,x')\,, \tag{5.8}$$

where $\mathcal{D}_{\mu\nu}$ is a differential operator determined from the lagrangian whose precise form is not important for the present purpose. We cannot take the bi-distribution $F(x,x')$ to be the Wightman function, as its singularity as $x \to x'$ would produce a divergent result for $\langle T_{\mu\nu}(x)\rangle$. We must first renormalize: this is done by defining

$$F(x,x') = G(x,x') - H(x,x')\,, \tag{5.9}$$

where $H(x,x')$ is the Hadamard bi-distribution. This has the form

$$H(x,x') = \frac{1}{(2\pi)^2}\frac{U(x,x')}{\sigma(x,x')} + V(x,x')\log\sigma(x,x') + W(x,x')\,, \tag{5.10}$$

where σ is half the square of the geodesic distance between x and x', and $U(x,x) = 1$. In the massive case, the functions U, V, W can be determined by requiring that H satisfy the Klein-Gordon equation in each argument. Thus, this renormalisation is state-independent, being entirely determined by the geometry of the spacetime manifold.

If $G(x,x')$ is of the Hadamard form, the subtraction in (5.9) will cancel the divergences in the two functions, allowing us to define a finite renormalised stress tensor via (5.8). However, in the case of interest, the Wightman function for an α-vacuum $G_\alpha(x,x')$ is not of Hadamard form. This subtraction will not then define a good renormalised stress tensor.

The procedure adopted in [30] to address this problem was to introduce a small element of state-dependence into the renormalisation procedure: when we are considering an α-vacuum in de Sitter space, for which

$$\lim_{x\to x'} G_\alpha(x,x') = N_\alpha^2 e^{\alpha+\alpha^*}\left(\frac{1}{(2\pi)^2}\frac{1}{\sigma(x,x')} + \bar{V}(x,x')\log\sigma(x,x') + \bar{W}(x,x')\right) \tag{5.11}$$

for some $\bar{V}$ and $\bar{W}$, then we define[7]

$$F(x,x') = G(x,x') - N_\alpha^2 e^{\alpha+\alpha^*} H(x,x')\,, \tag{5.12}$$

[7] There is a difference in notation between our paper and [30], so the α used there is not the same as the one used here.

where $H(x, x')$ is given by (5.10), and use (5.8) for this F to define the stress tensor. One can also think of this as defining $\langle T_{\mu\nu}\rangle[G_\alpha] = N_\alpha^2 e^{\alpha+\alpha^*}\langle T_{\mu\nu}\rangle[(N_\alpha^2 e^{\alpha+\alpha^*})^{-1}G_\alpha]$, where $\langle T_{\mu\nu}\rangle[(N_\alpha^2 e^{\alpha+\alpha^*})^{-1}G_\alpha]$ is defined by the usual procedure. The point is that $\bar{G} = (N_\alpha^2 e^{\alpha+\alpha^*})^{-1}G_\alpha$ is of Hadamard form. Although this procedure introduces state-dependence into the renormalisation procedure, the stress tensor so defined shares all the good properties of the usual stress tensor: the difference in energy for excited states above an α-vacuum will be given by the usual point-splitting expression; the construction remains local; and $\nabla^\mu\langle T_{\mu\nu}\rangle = 0$ (since the additional factor introduced is a constant). This allows us to discuss the renormalised stress tensor in the α-vacua on de Sitter space.

We can apply this same prescription to obtain a notion of $\langle T_{\mu\nu}\rangle$ for our α-vacuum states on the bubble of nothing or locally AdS black hole spacetimes. For the bubble of nothing, this gives a finite well-defined stress tensor everywhere. For the black hole, however, this prescription breaks down on the black hole horizon: the new singularities arising from the terms involving $G_E(x, x'_A)$ and $G_E(x_A, x')$ in (4.11) are not cancelled by the subtraction (5.12), so the stress tensor becomes ill-defined on the horizon. The new singularities imply that $\bar{G} = (N_\alpha^2 e^{\alpha+\alpha^*})^{-1}G_\alpha$ fails to be of Hadamard form on the horizon.

Thus, the ill-behavedness of the α-vacua on the horizon of the locally AdS black hole is signalled through the breakdown of the procedure of [30] for defining a renormalised stress tensor.

5.3 Analytic continuation

Another way to see the relation between the event horizon and α-vacua is to consider the extension of the analytic continuation argument of [31, 32] to this case. In [32], it was shown that an n-point correlation function in the CFT on the two boundaries of the eternal BTZ black hole could be related either to bulk interactions integrated over the region $r \geq r_+$ in the bulk spacetime or over the region $r \geq 0$, including the region behind the event horizon (with a different $i\epsilon$ prescription for the bulk-boundary propagators). The argument used analyticity properties of the n-point function, deforming the contour integral over $r \geq r_+$ to an integral over the euclidean spacetime by complexifying the time coordinate, and then rotating back to the real lorentzian section in Kruskal coordinates.

Despite the somewhat different structure of the spacetime in the locally AdS black hole, we can apply a similar argument here. If we consider an integral which initially runs over the region $r \geq 0$ in the lorentzian black hole solution

$$ds^2 = \left(1 + \frac{r^2}{l^2}\right) d\chi^2 + \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2\left[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (5.13)$$

we can continue to a euclidean solution by $\tau \to -i\vartheta$, giving

$$ds^2 = \left(1 + \frac{r^2}{l^2}\right) d\chi^2 + \left(1 + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2\left[d\vartheta^2 + \cos^2\vartheta(d\theta^2 + \sin^2\theta d\phi^2)\right]. \quad (5.14)$$

We then define a new radial coordinate by $r = 2ly/(1 - y^2)$, so

$$ds^2 = \frac{4l^2}{(1-y^2)^2}\left\{dy^2 + y^2[d\vartheta^2 + \cos^2\vartheta(d\theta^2 + \sin^2\theta d\phi^2)]\right\} + \frac{(1+y^2)^2}{(1-y^2)^2}d\chi^2. \quad (5.15)$$

We can recover the Kruskal coordinates of (2.9) by defining cartesian coordinates y^i $i = 1, \ldots, 4$ on the $(y, \vartheta, \theta, \phi)$ space, and analytically continuing $y^4 \to iy^0$. As in the BTZ case, the cartesian coordinates are restricted to the interior of the unit ball $(y^i)^2 < 1$; however, the exterior of the unit ball is isometric to the interior, so we can take the integral to run over all y^i if we divide by a factor of two. The factor of two is then used to convert the integral over all y^μ in the Kruskal coordinates (2.9) to an integral over $-1 < y^\mu y_\mu < 1$, covering the full black hole spacetime.

The above procedure is possible only in the euclidean vacuum. If we consider an α-vacuum on the black hole, it will not be possible to continue the integral in this way, as the α-vacuum does not define a regular propagator on the euclidean spacetime when we analytically continue $\tau \to -i\vartheta$. That is, the additional pole associated with the extra divergences on the event horizon in an α-vacuum will obstruct this kind of contour deformation argument. Thus, we see again that the α-vacuum runs into trouble when we try to look inside the black hole.

6. Conclusions

The relation between the bulk and boundary for these time-dependent spacetimes has several new and interesting features. The identification of the cosmological/acceleration horizons in the bulk with the de Sitter cosmological horizon in the boundary provides an example of a new way of relating thermodynamic behaviour in the bulk to the boundary. We believe this kind of relation should be very general, applying to any non-compact horizon encountered in the AdS/CFT correspondence. This identification provides new insight into the thermodynamic interpretation of horizons in spacetime, since the boundary interpretation for our bulk horizons strongly suggests that the relation $S = A/4G$ can be applied even to such non-compact horizons. This provides support for the very general connection between entropy and horizons advocated in [41, 42]. We have also argued that the area of the event horizon in the locally AdS black hole should not be interpreted thermodynamically. We argued this from a purely spacetime point of view, but it also seems a natural result from the CFT side, since we conjectured that the CFT dual is a vacuum state, where we see no role for an increasing entropy.

A central result of our paper was to show that the analogues of α-vacua for a free scalar field on the locally AdS black hole spacetime break down on the event horizon. Thus, these are not good quantum states on the full black hole solution. The unique regular invariant vacuum state on the locally AdS black hole is the euclidean vacuum. We regard this as evidence that there are no α-vacua in the strongly-coupled dual field theory, which lives on de Sitter space cross a circle. This provides support, from a very different perspective, for the view taken by some authors that α-vacua are not good states in an interacting field theory from the point of view of perturbation theory [20, 21, 24, 26]. This selection of the euclidean vacuum as a preferred state provides an interesting example of how the bulk spacetime picture can be used to study issues of quantum field theory on more general backgrounds.

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It would be interesting to investigate further the interpretation of these two spacetimes from the boundary point of view. In particular, the fact that the bubble of nothing solution exists only if the radius of the χ circle is less than a maximum value seems quite mysterious from the boundary point of view.

In fact, there is also another issue of interpretation which remains open at a purely spacetime level. In the asymptotically flat case, the bubble of nothing solution is interpreted as describing a non-perturbative instability of the Kaluza-Klein vacuum, that is, flat space with one spatial direction periodically identified [6]. In the case of a negative cosmological constant, the analogous interpretation of the bubble solution would be to regard it as describing a non-perturbative decay of this quotient of AdS, the locally AdS black hole. However, the fact that this background is itself time dependent (and even has an event horizon!) may complicate this interpretation. See [48] for further discussion of the interpretation of this bubble as describing a non-perturbative instability.

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